also appears to be reasonable, to calculate Poisson's ratio, a negative value for Poisson's ratio is obtained. This indicates that one or both values are probably incorrect. If the relationships between the four elastic properties are valid, then in order to make the data compatible, assuming that all of the discrepancy does *not* lie in the bulk modulus, the correct Young's modulus value for selenium must be less than 0.591×10^6 kg/cm^2, the value given in Table I.

Estimated Data. As will be shown later in Section 22 and in Table XIX, the ratio of Young's modulus to the shear modulus, Y/μ, is very nearly a constant and therefore may be quite reliably estimated for elements for which this ratio is unknown. This ratio and the value for the bulk modulus (experimental or estimated value) may be used to calculate the estimated value of Young's modulus by means of the following equation:

$$Y = [9 - 3(Y/\mu)]B, \tag{3.1}$$

which is obtained by simple algebra from Eqs. (II.1) and (II.2). The bulk modulus was used in the calculations since it has been determined experimentally for more elements than any of the other elastic properties. Furthermore, because of this reason it was felt that unknown values of the bulk modulus could be better estimated from the periodic relationships of the elements (see Section 6). The estimated values of Young's modulus for white, red, and black phosphorus, arsenic, rubidium, strontium, and cesium were calculated by using the experimental value of the bulk modulus (Table V) and the estimated value of Y/μ (Table XIX); and for technetium, osmium, polonium, francium, radium, actinium, and protactinium by using the estimated values of both the bulk modulus and Y/μ. The estimated values of Young's modulus for promethium, thulium, and lutetium were estimated from the plot shown in Fig. 2a. The value for europium was estimated to be equal to the mean value for barium and ytterbium, since europium is also known to be divalent in the metallic state[13]; thus these three elements are respectively the initial, end, and middle members of the divalent $4f$ series.[13]

[13] Europium and ytterbium have been considered to be divalent because most of their physical properties, such as the atomic volumes, metallic radii, melting and boiling points, heats of sublimation, compressibilities, and coefficients of expansion are more like those of the alkaline-earth metals, calcium, strontium, and barium, than those of the rare-earth metals. This is confirmed by magnetic data, which indicate that europium has a $4f^7$ configuration and ytterbium a $4f^{14}$ configuration; in both instances this leaves only two electrons remaining beyond the xenon core, and presumably they are in the $6s^2$ state. Thus, since barium has a $4f^06s^2$ configuration, these three elements are the first (barium), mid (europium), and end (ytterbium) members of the divalent $4f$ transition series. Values for most of the above-mentioned properties are given in the following sections; those which are not may be found in K. A. Gschneidner, Jr., "Rare Earth Alloys," pp. 3–66. Van Nostrand, Princeton, New Jersey, 1961.

4. Shear Modulus

The shear modulus is also known as the torsion modulus and modulus of rigidity. The values of the shear moduli are listed in Table II, and are shown in Fig. 3 for the elements of the fourth, fifth, and sixth periods of the Periodic Table. The shear modulus varies between 0.13×10^6 kg/cm^2 for potassium and 4.6×10^6 kg/cm^2 for diamond. The estimated value for francium, however, indicates that the lower limit should be extended to 0.0063×10^6 kg/cm^2. The shear modulus, like Young's modulus, has a marked dependence on the electronic configuration of the element (Fig. 3). The similar shape of Figs. 1 and 3 is not surprising since $Y \simeq 2.6\mu$ (see Section 22 and Table XIX). The maximum value of the shear modulus encountered in a given period is associated with the elements having the maximum number of unpaired d electrons. The minimum near the end of each period occurs for the elements that have an s^2p^1 configuration. The anomalous behavior of the fourth-period elements as compared with those in the fifth and sixth periods is again evident. The behavior of the elements in the second and third periods is similar to that of their cogeners in the fourth through sixth period; e.g., the shear modulus increases as one proceeds from the alkali metals to the alkaline-earth metals to the group IIIA elements.

The variation in the shear modulus for the rare earths is shown in Fig. 2b. The point for ytterbium was ignored in drawing the straight line. The value for ytterbium is low because ytterbium is divalent.[13]

Boron, Selenium, Ruthenium, and Rhenium. No experimental values of the shear modulus exist for boron, selenium, ruthenium, and rhenium. However, from the experimental values of the bulk modulus (Table V) and Young's modulus (Table I) it was possible to calculate the shear modulus. For boron, ruthenium, and rhenium reasonable values were obtained for both the shear modulus and Poisson's ratio. For selenium, on the other hand, a negative value was obtained for Poisson's ratio, indicating that the calculated value for the shear modulus is probably incorrect. Therefore, the value for selenium given here was estimated (see below).

Estimated Data. From an estimated value of the ratio of Young's modulus to the shear modulus (see Section 22 and Table XIX) and the known values of Young's modulus, the estimated shear modulus was calculated for scandium, selenium, and neptunium. Since Young's modulus for selenium is probably too large (see Section 3), the estimated shear modulus is probably too large also. The estimated shear moduli for white, red, and black phosphorus, arsenic, rubidium, strontium, technetium, cesium, osmium, polonium, francium, radium, actinium, and protactinium are based both on estimated values of Young's modulus and on the ratio Y/μ.